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Inverse bremsstrahlung absorption in large radiation fields during binary collisions—classical theory

G J PERT

Department of Applied Physics, University of Hull, Hull, UK

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Abstract. A simple method of calculating the inverse bremsstrahlung absorption coefficient in binary collisions is presented. The approach allows calculations to be made of the absorption at any field strength provided the field is classical and the electron is nonrelativistic and its thermal distribution function is isotropic. Calculations are made for electron/atom collisions interacting by a power low central force. The results are extended to discuss absorption in plasmas. The effects of electron–electron and inelastic collisions are also considered.

1. Introduction

The absorption of electromagnetic radiation by electrons during a collision is a well known phenomenon. The problem has been investigated by two different methods. The kinetic approach considers the behaviour of the electrons in a gas under the influence of an electromagnetic field (MacDonald 1966). The electron distribution function is calculated from the Boltzmann equation via an expansion in terms of u/v_{T} where u is the oscillation velocity, and v_{T} the thermal speed. The second approach calculates the absorption coefficient directly from the bremsstrahlung emission cross sections of the electrons (Bekefi 1966). Since bremsstrahlung is a single photon process this method is only valid if $mu_0^2 \ll \hbar\omega$, that is, the classical absorption energy/collision is less than the photon energy. It is easily shown that if the electron thermal energy is very much greater than $\hbar\omega$, so that the quantum nature of the absorption may be neglected, both approaches lead to the same equation for the distribution function (Zel'dovich and Raizer 1965)

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \epsilon}\left(\alpha f - 2\alpha\epsilon\frac{\partial f}{\partial \epsilon}\right) + \left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}} \tag{1}$$

where $(\partial f/\partial t)_{\mathrm{c}}$ takes into account energy losses in elastic and inelastic collisions and $\alpha = \frac{1}{3}\nu_{\mathrm{eff}}\epsilon_0$. The quantity ν_{eff} is the elastic collision frequency and ϵ_0 the oscillation energy of the electron in the field of peak intensity E_0. Equation (1) is valid up to third order in u/v_{T}.

If the field is very strong u/v_{T} may not be small and the results obtained from the small field perturbation theory may be in error. The effects of multiphoton bremsstrahlung have been considered by Rand (1964) and by Bunkin and Federov (1966); the results are not however in a very convenient form for manipulation. Hora (1970) has indicated classically that departures from the classical absorption formula for an ionized gas (Spitzer 1956) may be expected when $v_{\mathrm{T}} \lesssim u$.

In this paper a simple classical nonrelativistic method is indicated which enables the absorption of a group of electrons of constant thermal speed to be calculated. The method is limited in its applicability by the assumption of isotropy of the distribution function with respect to the thermal (time averaged) speed. As is well known the condition is obeyed to second order for $u/v_T \ll 1$; similarly by averaging over collisions it can also be shown to be valid to second order for $v_T/u \ll 1$.

This technique is thus limited to first order for the high and low field cases. To higher order a more refined technique must be used. However the results are obtained in an easily used form. If it can be shown that the distribution function is isotropic despite the field, the results will be valid over the whole range of v_T/u; for example in a fully ionized plasma absorption only occurs in heavy particle–electron collisions (in the dipole approximation), the electron collisions simply introducing a randomization of the electron velocity vector. In this case the distribution function will be isotropic and the exact results obtained for the absorption may be integrated over the electron distribution, which will approximate to a maxwellian to obtain the total absorption coefficient of the plasma.

In addition to the limits imposed by the distribution function, the electron is considered to move classically and nonrelativistically in a uniform, monochromatic wave field such that the oscillation frequency ω satisfies

$$\frac{1}{\tau} \gg \omega \gg \nu_{\text{eff}}$$

where τ is the duration of the collision. The classical approximation implies $mvc \gg \hbar\omega$, but the classical field approximation can be written $\frac{1}{2}mu^2 \gg \hbar\omega$ and is therefore a more restrictive condition, since the electrons are assumed nonrelativistic.

2. Thermal energy increase in heavy particle elastic collisions

In an oscillating electromagnetic field it is well known that electrons oscillate a quarter period out of phase with the field: thus the electron velocity due to the field is

$$\boldsymbol{u} = \boldsymbol{u}_0 \sin \omega t \tag{2}$$

where

$$\boldsymbol{u}_0 = \frac{e}{m\omega}\boldsymbol{E}_0$$

and e and m are the electron charge and mass respectively and ω the angular frequency of the field. In addition to this velocity the electron will also have its thermal velocity $\boldsymbol{v}_T$, giving a total velocity

$$\boldsymbol{v} = \boldsymbol{v}_T + \boldsymbol{u}. \tag{3}$$

The total electron energy is thus

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m\boldsymbol{u}^2 + \tfrac{1}{2}mv_T^2 + m\boldsymbol{u} \cdot \boldsymbol{v}_T. \tag{4}$$

It can be shown that averaging over all electron velocity directions

$$\overline{\boldsymbol{u} \cdot \boldsymbol{v}_T} = 0 \tag{5}$$

to second order for either $(v_T/u \ll 1)$ or $(v_T/u \gg 1)$ provided $\omega \gg \nu_{\text{eff}}$, the collision frequency. Physically this result is due to the loss of memory of the previous collision due to the oscillations between collisions, that is, the next collision occurs at a random part of the oscillation independent of the previous collision. Under these conditions the electron energy may be clearly divided into two parts, one the energy of oscillation $\frac{1}{2}m\boldsymbol{u}^2$ the other the random thermal energy $\frac{1}{2}mv_{\mathrm{T}}^2$.

The problem is to calculate the rate of thermal energy gain by electrons in the field. This may be done by calculating the thermal energy gain in a collision with given $\boldsymbol{u}$ and $\boldsymbol{v}_{\mathrm{T}}$; and then averaging over electrons, that is over $\boldsymbol{u}$ and $\boldsymbol{v}_{\mathrm{T}}$. The energy gain per collision is easily calculated once the nature of the collision is understood. Before the collision at a time t the electron has velocity

$$\boldsymbol{v} = \boldsymbol{v}_{\mathrm{T}} + \boldsymbol{u}(t). \tag{6}$$

After collision the velocity will be changed to

$$\boldsymbol{v}' = \boldsymbol{v}'_{\mathrm{T}} + \boldsymbol{u}(t').$$

The collision has thus changed the thermal velocity from $\boldsymbol{v}_{\mathrm{T}}$ to $\boldsymbol{v}'_{\mathrm{T}}$. If the collision takes a time very much less than $1/\omega$, then the velocity $\boldsymbol{u}$ at which the collision occurs is clearly defined (it should be noted that even in optical fields this condition will usually be obeyed).

The gain in thermal energy due to an electron/atom collision is thus

$$\tfrac{1}{2}mv_{\mathrm{T}}'^2 - \tfrac{1}{2}mv_{\mathrm{T}}^2 = m\boldsymbol{u} \cdot (\boldsymbol{v}_{\mathrm{T}} - \boldsymbol{v}'_{\mathrm{T}}) \tag{7}$$

provided the collision is elastic, and there is no energy transfer to the heavy particle, since the ratio of masses of the electron and heavy particle is very small. We therefore consider an assembly of electrons with differing thermal velocities, but all oscillating with the field. The electrons collide from time to time with the heavy particles, which may be regarded as stationary targets. During the collisions the electrons are heated, the energy so converted being identified as radiation absorbed.

In the time interval δt the number of collisions of electrons with velocity $(\boldsymbol{u} + \boldsymbol{v}_{\mathrm{T}})$ resulting in scattering through an angle θ into solid angle $\mathrm{d}\boldsymbol{\Omega}$/unit volume, is given by

$$I(\theta) n v f(\boldsymbol{v}_{\mathrm{T}})\, \delta t\, \mathrm{d}\Omega\, \mathrm{d}\boldsymbol{v}_{\mathrm{T}} \tag{8}$$

where $I(\theta)$ is the differential elastic collision cross section, n the density of heavy particles and $f(\boldsymbol{v}_{\mathrm{T}})$ the electron distribution function.

The total energy gain by electrons with thermal speed v_{T} and oscillation velocity $\boldsymbol{u}$ in δt/unit volume is

$$\tfrac{1}{2}mnf(v_{\mathrm{T}})\, \mathrm{d}v_{\mathrm{T}} \int I(\theta) v (v_{\mathrm{T}}'^2 - v_{\mathrm{T}}^2)\, \mathrm{d}\Omega\, \delta t. \tag{9}$$

This quantity must be integrated over the scattering solid angle and then averaged over all directions of $\boldsymbol{v}_{\mathrm{T}}$ (since the direction of $\boldsymbol{v}_{\mathrm{T}}$ is random, as discussed earlier) and values of $\boldsymbol{u}$.

Defining the scattering angles (θ, ϕ) with respect to incoming velocity $\boldsymbol{v}$ and the plane of $\boldsymbol{u}$ and $\boldsymbol{v}$

$$\begin{aligned} \tfrac{1}{2}(v_{\mathrm{T}}'^2 - v_{\mathrm{T}}^2) &= \boldsymbol{u} \cdot (\boldsymbol{v} - \boldsymbol{v}') \\ &= \boldsymbol{u} \cdot \boldsymbol{v}(1 - \cos\theta) - |\boldsymbol{u}|\,|\boldsymbol{v}| \sin\theta \cos\phi \sin\theta' \end{aligned} \tag{10}$$

where θ' is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. Integrating over θ and ϕ, the energy gain/electron for electrons of velocity $\boldsymbol{v}_{\mathrm{T}}$ and $\boldsymbol{u}$ in time δt is given by

$$mnv \int I(\theta)(1-\cos\theta)\boldsymbol{u}\cdot\boldsymbol{v}\,\mathrm{d}\Omega\,\delta t = mnv\boldsymbol{u}\cdot\boldsymbol{v}\sigma_{\mathrm{d}}\,\delta t \tag{11}$$

where σ_{d} is the momentum transfer cross section and is in general a function of v.

We next average over all angles χ between $\boldsymbol{u}$ and $\boldsymbol{v}_{\mathrm{T}}$ to give the average energy gain/electron of speed v_{T} in time δt as

$$mn(\overline{\boldsymbol{u}\cdot\boldsymbol{v}v\sigma_{\mathrm{d}}})\,\delta t. \tag{12}$$

3. 'Power law of force' collisions

This average cannot be performed unless the dependence of σ_{d} on v is known. Fortunately for many important cases this is given by a simple power law

$$\sigma_{\mathrm{d}} = \gamma v^{-4/(s-1)} \tag{13}$$

where s is the power of the central field force causing the scattering. Using this relation the average can be performed for values of s.

3.1. *Hard sphere*

If $s = \infty$ the collision cross section is a constant and equation (12) yields for the energy gain in δt/electron

$$\frac{4}{3}mn\sigma_{\mathrm{d}}u^2v_{\mathrm{T}}\,\delta t\left(1+\frac{u^2}{5v_{\mathrm{T}}^2}\right) \qquad v_{\mathrm{T}} > u \tag{14}$$

$$mn\sigma_{\mathrm{d}}u^3\,\delta t\left(1+\frac{2v_{\mathrm{T}}^2}{3u^2}-\frac{v_{\mathrm{T}}^4}{15u^4}\right) \qquad v_{\mathrm{T}} < u. \tag{15}$$

These results are exact, to the order of isotropy of the electron distribution.

3.2. *Maxwellian particles*

If $s = 5$ the collision frequency ν is a constant. It should be noted that this is an important practical case corresponding to classical polarizable molecules. The energy gain in unit time is

$$mn(\sigma_{\mathrm{d}}v)\boldsymbol{u}^2 = m\boldsymbol{u}^2\nu. \tag{16}$$

3.3. *Coulomb force law*

If $s = 2$ the momentum transfer cross section is given by

$$\sigma_{\mathrm{d}} = \gamma v^{-4}\ln\Delta \tag{17}$$

where Δ is a function of v (equations (27) and (29)). Neglecting the slow variation of the log term in the integration yields for the energy gain in unit time

$$0 \qquad v_{\mathrm{T}} > u \tag{18}$$

$$mn\gamma\frac{\ln\bar{\Delta}}{|u|} \qquad v_{\mathrm{T}} < u. \tag{19}$$

3.4. Asymptotic solutions for general values of s

In addition to the results above which are exact, asymptotic solutions for general values of $s(>2)$ can also be simply calculated. For the energy gain in unit time we have

$$mn\gamma v_T^{\beta}\boldsymbol{u}^2\left(1+\frac{\beta}{3}\right)+\ldots \qquad v_T \gg u \tag{20}$$

$$mn\gamma|\boldsymbol{u}|^{(2+\beta)}+\ldots \qquad v_T \ll u \tag{21}$$

where $\beta = 1-4/(s-1)$.

It is easily shown that the result for $v_T \gg u$ is also obtained by integrating (1).

These results may now be averaged over the oscillation of the electron to give the average energy gain of an electron of thermal speed v_T/unit time.

(i) For hard sphere molecules the energy gain/unit time is given by

$$\tfrac{2}{3}mnu_0^2\sigma_d v_T = \tfrac{2}{3}mu_0^2 v_{\text{eff}} \qquad v_T \gg u_0 \tag{22}$$

$$\frac{4}{3\pi}mnu_0^3\sigma_d \qquad v_T \ll u_0. \tag{23}$$

(ii) For maxwellian molecules the energy gain/unit time is given by

$$\tfrac{1}{2}mu_0^2\nu. \tag{24}$$

(iii) For the Coulomb force law the energy gain/unit time is given by

$$\frac{1}{\pi}mn\frac{mn\gamma\ln\bar{\Delta}}{u_0}\ln\left(\frac{u_0+(u_0^2-v_T^2)^{1/2}}{u_0-(u_0^2-v_T^2)^{1/2}}\right) \qquad v_T < u_0 \tag{25}$$

$$\frac{2}{\pi}\frac{mn\gamma\ln\bar{\Delta}}{u_0}\ln\left(\frac{2u_0}{v_T}\right) \qquad v_T \ll u_0. \tag{26}$$

The value of $\bar{\Delta}$ is obtained by noting that the major contribution for the average comes from $u \simeq v_T$: the contribution to the integral in (25) for $u < v_T$ being given by (18) and that for $u > v_T$ by (19).

For $v_T > u_0$ the average energy gain cannot be calculated in this simple manner, neglecting the behaviour of the ln term. Δ must be written explicitly in terms of v and the ln term expanded in terms of the small order term in u/v_T, which yields the average energy gain.

Coulomb force laws are found in practice in two important cases: high velocity atomic scattering (Born approximation) and ionic scattering. For the former

$$\sigma_d = 4\pi\left(\frac{Ze^2}{mv^2}\right)\ln\left(\frac{\gamma hv}{e^2}\right) \tag{27}$$

where γ is a constant characteristic of the atom. From equation (26) the photon absorption cross section in high fields is thus

$$\sigma = \frac{2^6\pi Z^2e^3n_i\omega}{cE_0^3}\ln\left(\frac{2u_0}{v_T}\right)\ln\left(\frac{2\gamma hv_T}{\pi e^2}\right) \qquad u_0 \gg v_T \tag{28}$$

which is in good agreement with results obtained directly from a quantum mechanical calculation (compare Rand (1964) for $\chi = \pi/2$ and Bunkin and Federov (1966) which contains a numerical error).

4. Absorption in a plasma

Although the principal collision mechanism resulting in bremsstrahlung absorption in a plasma is electron–ion collisions, these calculations cannot be directly applied to this case due to the long range nature of the force and electron correlations (Dawson and Oberman 1962). We may however still obtain qualitatively correct results in the case $\omega \gg \omega_p$ the plasma frequency, which are in error only by a slowly varying log factor of the order of 1 by the frequently used technique of a cut-off (Oster 1961) when the collisions are predominately binary. In very low frequency fields it is usually assumed that the upper cut-off can be taken at the Debye length d due to the shielding of the ion charge. However the requirement that the duration of the collision be small compared to the period of the field introduces a more stringent cut-off. Thus particles with impact parameter $b' \geqslant v_T/\omega$ must be excluded. Since $d = v_T/\omega_p$ and $\omega_p \ll \omega$; $b' = v_T/\omega < d$, and therefore b' rather than d must be used as the upper cut-off (Heald and Wharton 1965).

The field is also effectively cut-off on the short range side by the condition for the 90° scattering, namely the Landau parameter b^*. The result of these cut-offs is to introduce the log term

$$\ln\left\{1+\left(\frac{2b'}{b^*}\right)^2\right\}^{1/2} \simeq \ln\left(\frac{2b'}{b^*}\right) \tag{29}$$

into the momentum transfer cross section (Sutton and Sherman 1965, p 143). In a large radiation field the simple theory leading to the cut-off equation (29) no longer holds. The cut-offs in this case depend on whether the behaviour of the electrons is dominated by thermal or field effects. The characteristic parameter determining these effects will be the length

$$l = \frac{kT}{eE} \tag{30}$$

over which thermal effects dominate over those due to the field. Thus at 'not too high' fields the previous theory will hold out to a distance l and the cut-offs are b^* and the smaller of l and b'. However at high fields $l < b^*$ and this model clearly no longer holds. In this case the minimum cut-off is clearly l and the maximum is determined by the maximum distance over which the field induced oscillation causes a collision, namely the amplitude of the oscillation

$$L = \frac{eE_0}{m\omega^2}. \tag{31}$$

Thus the cut-offs are

$$\begin{array}{ll} b':b^* & b' < l \\ l:b^* & b^* < l < b' \\ L:l & b^* > l. \end{array} \tag{32}$$

For very fast particles the lower cut-off is determined by the electron wavelength $\lambda(=h/mv)$ where $\lambda > b^*$ or l, whichever is appropriate (Bekefi 1966).

At low fields the term not involving the log is zero (equation (18)). Since the log term is determined solely by the cut-off terms (equation (29)) it cannot be expected that a calculation of the absorption coefficient obtained as a result of the variation of the log term with the oscillation will yield accurate results when the effects treated by the cut-offs are so crudely represented. Such is indeed the case but the absorption coefficient calculated in this manner is in error by a factor of only 0·23 that obtained by more accurate methods (Scheuer 1960, Dawson and Oberman 1962)

However at high fields the absorption is not due to the log term, which is in fact averaged, and in consequence the results may be expected to be more accurate.

At high fields the absorption coefficient† is

$$k = \frac{32\pi Z^2 e^3 n_i n_e \omega}{c E_0^3} \ln\left(\frac{Ze^2 E_0^2}{m\omega^2 kT}\right) \ln \bar{\Delta} \qquad u_0 \gg v_T \tag{33}$$

where

$$\bar{\Delta} = \frac{l}{b^*} = \frac{kT/eE_0}{Ze^2/kT} = \frac{(kT)^2}{Ze^3 E_0} \qquad l > b^* \tag{34}$$

$$= \frac{L}{l} = \frac{eE_0/m\omega^2}{kT/eE_0} = \frac{(eE_0)^2}{m\omega^2 kT} \qquad l < b^*. \tag{35}$$

It must be appreciated that this calculation is likely to be only approximate in the $\ln \bar{\Delta}$ term due to the qualitative treatment of the electron collective effects.

5. Thermal energy increase in electron–electron collisions

In this case we consider the case of two oscillating electrons; with initial velocities at collision $\boldsymbol{v}_1 + \boldsymbol{u}$ and $\boldsymbol{v}_2 + \boldsymbol{u}$. After the collision their velocities are $\boldsymbol{v}'_1 + \boldsymbol{u}$ and $\boldsymbol{v}'_2 + \boldsymbol{u}$. Since the collision is elastic

$$\tfrac{1}{2}m(v_1^2 + \boldsymbol{u}^2) + \tfrac{1}{2}m(v_2^2 + \boldsymbol{u}^2) = \tfrac{1}{2}m(v_1'^2 + \boldsymbol{u}^2) + \tfrac{1}{2}m(v_2'^2 + \boldsymbol{u}_2^2) \tag{36}$$

and momentum is conserved

$$m(\boldsymbol{v}_1 + \boldsymbol{v}_2) = m(\boldsymbol{v}'_1 + \boldsymbol{v}'_2). \tag{37}$$

The change in thermal energy is

$$\tfrac{1}{2}m(\boldsymbol{v}_1'^2 + \boldsymbol{v}_2'^2 - \boldsymbol{v}_1^2 - \boldsymbol{v}_2^2) = m\boldsymbol{u} \cdot (\boldsymbol{v}'_1 + \boldsymbol{v}'_2 - \boldsymbol{v}_1 - \boldsymbol{v}_2) = 0. \tag{38}$$

There is thus no energy increase following an electron–electron collision, as may be expected since a system electron–electron has no dipole moment.

6. Thermal energy gain in inelastic collisions

In addition to the elastic collisions with the heavy particles, the electrons will also make inelastic collisions resulting in an energy transfer E_n to the nth excited state of the particle. In this case the thermal energy gain per second in collisions with the electron velocity $\boldsymbol{u}$ and $\boldsymbol{v}_T$ is

$$mnv \int \left(1 - \frac{|\boldsymbol{v}'|}{|\boldsymbol{v}|} \cos\theta\right) \boldsymbol{u} \cdot \boldsymbol{v} I_n(\theta)\, d\Omega \tag{39}$$

† A calculation of this term has also been reported by Babuel-Peyrissac (1970) using the Dawson–Oberman procedure, which also shows the difference between the regions $l < b^*$ and $l > b^*$.

where θ is the scattering angle. $I_n(\theta)$ is the differential cross section for excitation of the nth state and $\boldsymbol{v}'$ the final velocity is given by

$$v'^2 = v^2 - \frac{2E_n}{m}. \tag{40}$$

Further progress can only be made analytically in the case where the electron velocity $v \gg (2E_n/m)^{1/2}$. In this case the integral over θ can be performed and the result summed over all states of the particle to yield the total thermal energy gain per second by an electron of velocity $\boldsymbol{u}$, $\boldsymbol{v}_{\mathrm{T}}$

$$mnv\left(\boldsymbol{u}\cdot\boldsymbol{v}\sigma_{\mathrm{in,d}} - \frac{K}{m}\right) \tag{41}$$

where K is the retardation (Landau and Lifshitz 1958) and $\sigma_{\mathrm{in,d}}$ is the total inelastic diffusion cross section.

If the velocity is sufficiently fast the cross sections may be calculated by the Born approximation to yield

$$\begin{aligned} \sigma_{\mathrm{in,d}} &= \gamma' v^{-4} \ln \Delta' \\ K &= \beta v^{-2} \ln \delta \end{aligned} \tag{42}$$

where Δ' and δ are dependent on the velocity.

Performing the integrals as before we obtain for the average thermal gain per second by an electron of speed u and thermal velocity v_{T}

$$0 - \frac{1}{2} n\beta \ln \frac{\bar{\delta}}{v_{\mathrm{T}}} \qquad v_{\mathrm{T}} > u \tag{43}$$

$$\frac{mn\gamma' \ln \bar{\Delta}' - \frac{1}{2} n\beta \ln \bar{\delta}}{|u|} \qquad v_{\mathrm{T}} < u. \tag{44}$$

For $v_{\mathrm{T}} > u$ an average value has been taken for $\ln \bar{\Delta}'$ leading to the zero, as in elastic collisions (equation (18)): more correctly the variation of the log term should be taken into account. In this case since $\Delta' \simeq v$ we obtain for $v_{\mathrm{T}} \gg u$, the average thermal energy gain per second per electron as

$$\frac{1}{6}\frac{mnu_0^2\gamma'}{v_{\mathrm{T}}^3} - \frac{1}{2}\frac{n\beta \ln \bar{\delta}}{v_{\mathrm{T}}} \qquad v_{\mathrm{T}} \gg u \tag{45}$$

and for $u \gg v_{\mathrm{T}}$, the average thermal energy gain per second per electron as

$$\frac{2}{\pi}\frac{n}{u_0}\left[m\gamma' \ln \bar{\Delta}' \ln\left(\frac{2u_0}{v_{\mathrm{T}}}\right) - \beta \ln \bar{\delta}' \left\{\ln\left(\frac{2u_0}{v_{\mathrm{T}}}\right) + 1\right\}\right]. \tag{46}$$

7. Elastic energy losses

In a collision with a stationary gas molecule the total electron energy is not left unchanged but is decreased by a factor $(2m/M)(1 - \cos\theta)$ where M is the mass of the molecule. This factor can be taken into account in a similar manner to that used to investigate inelastic losses.

The electron thermal energy gain per second for electrons of velocity $\boldsymbol{v}$ and oscillation $\boldsymbol{u}$ is

$$mnv\left(\boldsymbol{u}\,.\,\boldsymbol{v}-\frac{m}{M}v^2\right)\sigma_{\mathrm{d}}$$

since $m/M \ll 1$.

8. Discussion

These results for the elastic scattering confirm the $1/E_0^3$ dependence of the absorption coefficient found previously. It can be clearly seen that this dependence results from the characteristic $1/v^4$ dependence of the cross section given by the Born approximation at large velocities. The $1/E_0$ dependence found by Hughes and Nicholson-Florence (1968) can be seen to be due to the time average of the instantaneous power absorption being performed using the radiation field instead of the electron velocity; when this is modified the dependence is again found to be $1/E_0^3$.

A similar problem has also been considered by Afanas'ev *et al* (1970) for atomic collisions. In this work the distribution function is calculated from equation (1) taking into account energy losses due to inelastic collisions, the energy gain in inelastic collisions which will be of the same order as that in elastic collisions being neglected. Since at high electron energy ($\simeq 100$ eV) inelastic collisions in fact become more probable than elastic, the neglect of this term may cause significant error when

$$\frac{e^2E_0^2}{2m\omega^2} > I$$

where I is the ionization energy. Under these conditions the electron has sufficient energy to ionize the atom in nearly each collision. These collisions may be included using equation (46) provided the distribution function is known and is isotropic.

In the calculation of the absorption in atomic collisions it has assumed that the collision cross section of the electron on the atom is known. Clearly since this term depends on the structure of the atom involved in the collision the cross section itself may be modified by the field. At low fields the atom will be only slightly perturbed and the cross section for collisions will be approximately that for zero field. In high fields the electron velocities are high and the cross section is given by the Born approximation. As the elastic cross section (equation (27)) contains the atomic structure only through the term γ which appears in the logarithm, it can be seen that little error will be incurred in using the zero field cross section. In the case of inelastic collisions, however, involving transitions to highly perturbed excited states, it is not obvious that the zero field cross section can be used. This point will be discussed in a subsequent paper and it will be shown that provided the sum is taken over all states of the atom (excluding the ground state but including the continuum) the total inelastic cross section is that calculated in zero field.

It will be appreciated that the collision frequency for electron–electron collisions will be determined by the thermal velocity $\boldsymbol{v}_{\mathrm{T}}$, whereas that of electron–heavy particles will correspond to the total velocity $\boldsymbol{v} = \boldsymbol{v}_{\mathrm{T}}+\boldsymbol{u}$. Thus in very high fields where the oscillation velocity is high the electron–heavy particle collision frequency will be decreased by the inverse fourth power dependence of the cross section on velocity. As a result it must be expected that those transport coefficients which depend on the electron–heavy particle collision frequency (electron–heavy particle relaxation, electrical and thermal conduction, diffusion etc) will be greatly modified by the field. On the other hand effects resulting solely from electron–electron collisions (electron–electron relaxation etc) will not be changed. As a result it may be expected that the electron distribution function in an ionized gas will be maxwellian despite any anisotropy introduced by the bremsstrahlung.

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